The Aggregate Effects of Firm-Level Trade Distortions*

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Abstract

Trade models typically assume that fixed and variable trade costs are constant across firms. Using a model of firm-level trade, we show how to use a small number of empirical moments to compute model-implied measures of dispersion in trade distortions across firms. We then show how to relate changes in the dispersion of trade distortions to changes in aggregate income and trade volumes. We apply these techniques to firm-level data from China and France. We measure the effect of reducing the dispersion of export distortions from that of all Chinese firms (including state-owned, multinational, and the variety of other legal classifications) to that of private Chinese firms, and the effect of reducing the dispersion of export distortions for Chinese firms to that of French firms. We find sizable effects on aggregate income and total trade, which implies that the *dispersion* of trade distortions across firms (and not just the level of trade distortions) is important for welfare.

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1 Introduction

A fundamental question in international trade is how access to foreign markets affects aggregate efficiency. The benefits from trade are reduced when trade distortions limit access to foreign markets, and a central question in trade is the magnitude of the benefits to reducing these distortions. However, typically these distortions are assumed to be constant at the industry, market or even country level. Less attention has been focused on the case were firms within a given country vary in their fixed and variable costs of accessing foreign markets, as would be the case if governments give subsets of firms easier and cheaper access to export markets (such as targeted export loans, access to special economic zones, and preferred access to foreign exchange) or more difficult or expensive access to export markets (such as capital controls, or export licenses). These distortions across markets for firm output are analogous to the type of distortions in variable inputs considered in Hsieh and Klenow (2009).

In this paper we consider a model of trade in which monopolistically competitive firms choose whether or not to enter export markets, as in Melitz (2003), and are heterogeneous along three dimensions: productivity, variable trade costs and fixed trade costs. In this environment, we first derive a model-implied measure of the dispersion of variable and fixed trade cost distortions that is easy to measure using firm-level data, and is directly related to welfare. Second, we show how to relate changes in the dispersion of both fixed and variable trade cost distortions to real income and aggregate trade volumes.

The measurement of each distortion is intuitive. Dispersion in variable cost distortions is identified from dispersion in the ratio of export sales to domestic sales in the data. Since the model has households that aggregate varieties with a constant elasticity of substitution, the undistorted, efficient outcome is for the ratio of export sales to domestic sales to be the same for all firms. Therefore, dispersion in the proportion of sales across markets measures the magnitude of variable cost distortions.[1] We measure

[1] It is important to note that this argument ignores other sources of variation in the proportion of sales across markets. For example, firm-level demand shocks that are imperfectly correlated across markets would imply that it is efficient for the proportion of sales across markets to vary by firm. For this reason we do not consider the extreme case where all distortions are removed. In our measures of variable cost dispersion we first control for observables, then consider changes in residual dispersion between sets of firms (comparing Chinese firms to French firms, and comparing groups of firms within China).

dispersion in fixed trade costs using the relationship between export status and domestic sales. In an undistorted economy there is a sharp productivity threshold above which all firms export and below which no firms export. Dispersion in fixed trade costs weakens the correlation between productivity and export status as some low productivity firms face low fixed costs (and choose to export) and some high productivity firms face high fixed cost (and choose not to export). We derive a model-implied summary statistic for the distribution of fixed trade distortions can be measured analytically without making a functional form assumption for the distribution of these unobserved fixed costs. This summary statistic depends on the correlation between domestic sales and an indicator function for export status. When this correlation is high it implies that the relationship between productivity and export status is strong, which implies there is little dispersion in fixed trade costs. We then provide results that relate these measures directly to welfare.

We consider two empirical examples to measure gains from reducing trade cost dispersion. First, the Chinese government conducts aspects of its trade policy by targeting particular firms to receive preferred access to foreign markets or foreign exchange. One way that these policies are implemented is through a complex system of firm classifications that allow firms to face differing regulatory regimes.[2] This system implies that some firms face higher or lower variable and fixed trade costs than do others, which distorts export decisions. We first demonstrate that our measures of the dispersion in both fixed and variable cost distortions are lower among private firms than in the set of firms overall, and we measure the effect of changing the distribution of trade distortions in the set of firms overall to that of Chinese firms. Changing the variable trade cost distribution increases real income by 0.81% and total trade by 27.03%. However, changing the fixed trade cost distribution has very small effects: this causes a 0.04% increase in real income and a 1.20% increase in the volume of trade.

In our second example, we show that French firms exhibit less dispersion in both variable and fixed cost distortions than do Chinese firms. The difference in our model-implied measure of dispersion between Chinese and French firms is large. Changing

[2] The set of firm classifications is large, but includes state-owned, collective, and private.

the variable trade cost distribution of Chinese firms to that of French firms implies a 10.34% increase in real income and a 303.76% increase in total trade. For fixed costs, changing the Chinese distribution to the French implies a 1.58% increase in real income and a 52.21% increase in total trade. These effects are large, since application of the familiar formula from Arkolakis, Costinot and Rodriguez-Clare (2012) implies the gain from autarky to observed trade is 2.93%. Our view is that this is proof that heterogeneous trade distortions across firms are an issue of first order importance for research in trade.

All of these results are obtained with minimal data and computational requirements. In particular, all of our results can be computed given an aggregate trade elasticity, two aggregate statistics and four firm-level moments that can be easily computed from any firm-level data set with firm-level domestic and export sales.[3]

This paper is very much in the spirit of the large, recent literature on misallocation and distortions in the style of Restuccia and Rogerson (2008) and Hsieh and Klenow (2009). While those papers focus on distortions across firms in input markets, we study misallocation across output markets. In addition, our environment features distortions to the discrete cost to export. This is analogous to the literature studying distortions that affect firm entry choices, such as Asturias et al. (2017), Poschke (2010) and Herrendorf and Teixeira (2011). The interaction of export choices with firm-level distortions is studied in Vivalt (2012). Our novel contribution is developing an intuitive, model-implied approach to measure the magnitudes of export fixed cost distortions, and relating changes in these distortions to aggregate outcomes.

Our paper is directly related to the literature studying higher moments of the tariff schedule. Anderson and Neary (1996) and Feenstra (1995) study a model-implied measure of tariff distortions and show that variance in tariffs has a strongly distortionary effect on trade. This was extended to allow for extensive margin adjustment by Anderson and Neary (2007). Our exercise is similar, but focused on distortions to firms rather than industries, and allows for other firm-level distortions beyond tariffs.

[3] The two aggregate statistics are the import penetration ratio and the domestic country's share of world gross domestic product. The firm-level moments are the fraction of firms that export, the correlation of export status with domestic sales, the coefficient of variation of domestic sales, and a model-implied measure of dispersion in export intensity (see Proposition 2). In the special case where the elasticity of substitution across products is equal to 2, this measure of dispersion in export intensity is equal to one plus the square of the coefficient of variation in export intensity.

Finally, we build on a body of work studying particular types of distortions that affect export behavior, such as financial frictions in Kohn, Leibovici and Szkup (2016) or anti-dumping policy in Ruhl (2014). The strength of our approach relative to these is our generality. Our results make no assumptions about the source of the underlying heterogeneity, and so can be applied generally. The strength of the other approaches is precisely their specificity: by identifying the source of distortions across firms they have sharper implications for policy.

2 Model

There are two countries that we refer to as the home country and the foreign country respectively. Each country has a representative household that consumes products from both countries. Each product is made by a single firm, so the labels for firms and products are interchangeable. The set of firms that sell to country i is S_i. The household in country i solves:

$$P_i Y_i = \min_{\{y_i(j)\}} \int_{S_i} p_i(j) y_i(j) dj, \tag{2.1}$$

$$\text{subject to: } Y_i = \left(\int_{S_i} y_i(j)^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}}.$$

Cost minimization implies that the demand for product j in country i is given by:

$$y_i(j) = Y_i \left(\frac{P_i}{p_i(j)} \right)^{\sigma}. \tag{2.2}$$

In each country there is a unit mass of potential entrants. Potential entrant j in country i observes its productivity $z_i(j)$ and its export fixed cost distortion $\eta_i(j)$. It then decides whether or not to pay f^d units of labor to operate domestically, and whether or not to pay $f_i^x \eta_i(j)$ units of labor to export. Here, f_i^x is the physical cost of accessing the export market, and $\eta_i(j)$ is the fixed cost distortion. We interpret $\eta_i(j)-1$ as a distortionary tax, so that it can be easily interpreted as a policy parameter rather than a feature of the physical environment.[4] After paying the export fixed cost,

[4] If $\eta_i(j) < 1$, then it is a subsidy.

the firm observes its variable trade cost distortion $t_{i,k}(j)$ for $k \neq i$.[5] Firms face no variable cost distortion to sell in their home market, so $t_{i,i}(j) = 1$. Firms also pay an iceberg trade cost $\tau_{i,k}$ when exporting abroad, but none at home, so that $\tau_{i,i} = 1$.

Firm j that is based in country i solves the following problem when selling to country k:

$$\pi_{i,k}(j) = \max_{y_{i,k}(j), p_{i,k}(j)} p_{i,k}(j) y_{i,k}(j) - w_i \frac{y_{i,k}(j)}{z_i(j)} \tau_{i,k} t_{i,k}(j), \tag{2.3}$$
$$\text{subject to: } y_{i,k}(j) = Y_k \left(\frac{P_k}{p_{i,k}(j)} \right)^{\sigma}.$$

Solving the firm's problem implies optimal decision rules given by:

$$y_{i,k}(j) = Y_k P_k^{\sigma} \left(\frac{\sigma - 1}{\sigma} \frac{z_i(j)}{w_i \tau_{i,k} t_{i,k}(j)} \right)^{\sigma}, \tag{2.4}$$

so that firm sales to a given market are given by:

$$p_{i,k}(j) y_{i,k}(j) = Y_k P_k^{\sigma} \left(\frac{\sigma - 1}{\sigma} \frac{z_i(j)}{w_i \tau_{i,k} t_{i,k}(j)} \right)^{\sigma - 1}. \tag{2.5}$$

Now we define the value of entering as:

$$V_i(j) = \pi_{i,i}(j) - w_i f^d + \max\{E[\pi_{i,k}(j)] - w_i f_i^x \eta_i(j), 0\} \tag{2.6}$$

for $k \neq i$. A firm chooses to pay the fixed cost to operate if and only if $V_i(j) \geq 0$.

2.1 Firm-Level Distortions and Productivity

Next we describe the assumptions that we make about the distributions of productivity $z_i(j)$, fixed cost distortions $\eta_i(j)$, and variable cost distortions $t_{i,k}(j)$.

Following Chaney (2008), we require that productivity follows a Pareto distribution. In our context, this will allow us to make use of the tools we later use from

[5] To be clear, here we are assuming that firms do not observe their variable trade cost distortion until after they have paid to enter the export market. An alternative is to assume that variable trade distortions are observed before paying the export fixed costs, which implies a margin of misallocation where low productivity, low variable trade cost firms choose to export. This alternative is presented in Appendix C, where we show how this alternative affects our results. Our main results are unchanged. The reason we do not consider it as our base case is that it implies that low productivity exporters should be more export-intensive than high productivity exporters on average. In both data sets discussed in Section 5 there is no significant relationship between export intensity and either productivity or size. This motivates the timing assumption presented here.

Arkolakis, Costinot and Rodriguez-Clare (2012). We also assume that entry is interior in the sense that the fixed cost of entry is high enough that a firm drawing the lowest possible productivity does not enter. Furthermore, we require that the shape parameter of the Pareto distribution be large compared to the elasticity of substitution between products σ, so that the integrals we use to compute aggregates are guaranteed to exist. These are summarized in Assumption 1.

Assumption 1. *The distribution of productivity $z_i(j)$ satisfies:*

1. *Pareto distributed: productivity follows a Pareto distribution with shape parameter θ and scale parameter z_{min},*
2. *Regularity condition: $\theta > \sigma - 1 > 0$,*
3. *Interior entry choice: z_{min} is low enough that a potential entrant with productivity z_{min} does not enter.*

We model both fixed and variable trade distortions as taxes and subsidies. In order to satisfy aggregate resource feasibility we assume that the distortions are budget balanced. If S_i^k is the set of firms from country i that sell in country k, then budget balance implies that:

$$0 = \int_{S_i^k} w_i \frac{y_i(j)}{z_i(j)} \tau_{i,k}(t_{i,k}(j) - 1)dj. \tag{2.7}$$

Substituting in equation (2.4), this implies:

$$0 = \int_{S_i^k} Y_k P_k^\sigma \left(\frac{\sigma - 1}{\sigma}\right)^\sigma \left(\frac{w_i \tau_{i,k}}{z_i(j)}\right)^{1-\sigma} (t_{i,k}(j)^{1-\sigma} - t_{i,k}(j)^{-\sigma})dj. \tag{2.8}$$

Because $t_{i,k}(j)$ is drawn after the export fixed costs are paid, we assume that the variable trade distortion is independent of firm productivity and the fixed cost distortion. When exports are positive, budget balance is only satisfied when:

$$0 = \int_{S_i^k} t_{i,k}(j)^{1-\sigma} - t_{i,k}(j)^{-\sigma} dj \implies \int t_{i,k}^{-\sigma} dF_i(t_{i,k}) = \int t_{i,k}^{1-\sigma} dF_i(t_{i,k}) \tag{2.9}$$

These assumptions are summarized in Assumption 2.

Assumption 2. *The variable trade distortions $t_{i,k}(j)$ have cumulative density function F_i satisfy:*

1. *Independence: F_i is independent of productivity z or fixed cost distortion η,*
2. *Budget balanced: $\int t_{i,k}^{-\sigma} dF_i(t_{i,k}) = \int t_{i,k}^{1-\sigma} dF_i(t_{i,k})$.*

For fixed cost distortions, we assume that the productivity threshold that determines export status is interior to the set of operating firms at every value of the fixed cost. This implies that the distribution of fixed cost distortions is bounded away from zero.[6] That is, we assume there is no fixed cost distortion so low that every firm that draws it exports.

As with the variable trade distortion, we assume that the fixed cost distortions satisfy budget balance. We can derive the budget balance condition by noting that a firm enters the export market if and only if:

$$w_i f_i^x \eta_i(j) \leq \int \pi_{i,k}(z_i(j), t_{i,k}) dF_i(t_{i,k}) = \frac{Y_k P_k^\sigma}{\sigma} \left(\frac{z_i(j)}{\sigma} \frac{\sigma - 1}{w_i \tau_{i,k}} \right)^{\sigma-1} \int t_{i,k}^{1-\sigma} dF_i(t_{i,k}). \tag{2.10}$$

Therefore, for every η, there exists a cutoff $\bar{z}(\eta)$ above which all firms enter the export market and below which no firms enter. This is given by:

$$\bar{z}(\eta) = \left(\frac{w_i f_i^x \eta}{E\left[t_{i,k}^{1-\sigma}\right]} \frac{\sigma}{Y_k P_k^\sigma} \right)^{\frac{1}{\sigma-1}} \frac{\sigma}{\sigma - 1} w_i \tau_{i,k}. \tag{2.11}$$

Given Assumption 1, and assuming that η follows some distribution G_i, we dispense with firm indices and characterize firms by their productivity and fixed cost distortion to write the budget balance condition as:

$$0 = \int \left[\int_{\bar{z}(\eta)}^{\infty} w_i f_i^x (\eta - 1) \theta z_{min}^\theta z^{-\theta-1} dz \right] dG_i(\eta). \tag{2.12}$$

Substituting in equation (2.11) and rearranging terms implies:

$$0 = \int \eta^{\frac{\theta}{1-\sigma}+1} - \eta^{\frac{\theta}{1-\sigma}} dG_i(\eta) \implies E\left[\eta^{\frac{\theta}{1-\sigma}+1}\right] = E\left[\eta^{\frac{\theta}{1-\sigma}}\right]. \tag{2.13}$$

[6] In Appendix B we give an example where the Pareto distribution is convenient to work with. None of the results in this paper depend on a specific distributional form for the fixed cost distortions.

All assumptions made about the distribution of fixed costs are summarized below.

Assumption 3. *Export fixed cost distortions $\eta_i(j)$ follow a distribution characterized by cumulative density function G_i that satisfies:*

1. *Budget balanced: $\int \eta^{\frac{\theta}{1-\sigma}} dG_i(\eta) = \int \eta^{1+\frac{\theta}{1-\sigma}} dG_i(\eta)$,*
2. *Interior export choice: the support of G_i has a lower bound $a > 0$ such that there exists an entrant with $\eta_i(j) = a$ that chooses to operate, but does not pay the cost to export.*

2.2 Definition of Equilibrium

It is useful to summarize the model and assumptions made above by formally defining an equilibrium in this economy, even though the definition is standard.

We first rewrite several previously discussed conditions taking into account the assumptions made above. In particular, we now replace firm subscripts with the parameters that characterize all their production choices: productivity z, variable distortion t, and fixed cost distortion η.

The cutoff to enter the domestic market is given by the value of $z_{i,d}$ that solves:

$$\pi_{i,i}(z_{i,d}) = w_i f^d \tag{2.14}$$

Taking into account that the choice to pay the fixed cost to enter follows a cutoff rule, the household's cost minimization problem can be rewritten, for $k \neq i$, as:

$$\begin{aligned} P_i Y_i = \min_{\{y_{i,i}(z), y_{k,i}(z,t_{k,i})\}} & \int_{z_{i,d}}^{\infty} p_{i,i}(z) y_{i,i}(z) \theta \left(\frac{z_{min}}{z}\right)^{-\theta-1} dz \\ & + \int\int \left[\int_{\bar{z}_k(\eta)}^{\infty} p_{k,i}(z,t) y_{k,i}(z,t) \theta \left(\frac{z_{min}}{z}\right)^{-\theta-1} dz\right] dG_k(\eta) dF_k(z), \end{aligned} \tag{2.15}$$

$$\begin{aligned} \text{subject to: } Y_i^{\frac{\sigma-1}{\sigma}} = & \int_{z_{i,d}}^{\infty} y_{i,i}(z)^{\frac{\sigma-1}{\sigma}} \theta \left(\frac{z_{min}}{z}\right)^{-\theta-1} dz \\ & + \int\int \left[\int_{\bar{z}_k(\eta)}^{\infty} y_{k,i}(z,t)^{\frac{\sigma-1}{\sigma}} \theta \left(\frac{z_{min}}{z}\right)^{-\theta-1} dz\right] dG_k(\eta) dF_k(z). \end{aligned}$$

Labor market clearing in this environment, for $k \neq i$, is given by:

$$L_i = \int_{z_{i,d}}^{\infty} \left[f^d + \frac{y_{i,i}(z)}{z} \right] \theta \left(\frac{z_{min}}{z} \right)^{-\theta-1} dz \\ + \int \int \left[\int_{\bar{z}_i(\eta)}^{\infty} \left(f_i^x + \tau_{i,k} \frac{y_{i,k}(z,t)}{z} \right) \theta \left(\frac{z_{min}}{z} \right)^{-\theta-1} dz \right] dG_i(\eta) dF_i(z). \quad (2.16)$$

Trade is balanced, so for $k \neq i$:

$$\int \int \left[\int_{\bar{z}_k(\eta)}^{\infty} p_{k,i}(z,t) y_{k,i}(z,t) \theta \left(\frac{z_{min}}{z} \right)^{-\theta-1} dz \right] dG_k(\eta) dF_k(z) \\ = \int \int \left[\int_{\bar{z}_i(\eta)}^{\infty} p_{i,k}(z,t) y_{i,k}(z,t) \theta \left(\frac{z_{min}}{z} \right)^{-\theta-1} dz \right] dG_i(\eta) dF_i(z). \quad (2.17)$$

Then an equilibrium is, for each country i, the aggregate prices (w_i, P_i), aggregate output Y_i, entry productivity cutoff $z_{i,d}$, export productivity cutoff function $\bar{z}_i(\eta)$, and firm decision rules for each market k, $(p_{i,k}(z, t_{i,k}), y_{i,k}(z, t_{i,k}))$, that solve equations (2.11), (2.14), (2.15), (2.16) and (2.17), and solve the firm's problem defined in (2.3).

3 Analytical Results

In this section we measure the effect of changing both fixed and variable trade distortions. First, for any change in the distribution of trade distortions, we show how to compute a welfare-equivalent change in iceberg transportation costs. Second, we show how to relate any change in the distribution of trade distortions to aggregates so that the effect on real income and total trade can be computed.

3.1 Welfare-Equivalent Changes in Iceberg Trade Costs

We begin by computing welfare-equivalent changes in iceberg trade costs for any change in the distribution of fixed and variable trade distortions. It is useful to start with the formula for aggregate exports. Let $X_{i,k}$ be the value of goods consumed in country k and produced in country i. From equation (2.5):

$$X_{i,k} = \int \int \left[\int_{\bar{z}(\eta)}^{\infty} Y_k P_k^{\sigma} \left(\frac{z}{\sigma} \frac{\sigma - 1}{w_i \tau_{i,k} t} \right)^{\sigma-1} \theta z_{min}^{\theta} z^{-\theta-1} dz \right] dG_i(\eta) dF_i(t). \quad (3.1)$$

Integrating over z and rearranging implies:

$$X_{i,k} = \frac{\theta f_i^x z_{min}^\theta (\sigma-1)^\theta}{\theta - \sigma + 1} (\sigma w_i)^{1-\frac{\sigma\theta}{\sigma-1}} \left(\frac{Y_k P_k^\sigma}{f^x}\right)^{\frac{\theta}{\sigma-1}} \tau_{i,k}^{-\theta} E\left[t^{1-\sigma}\right]^{\frac{\theta}{\sigma-1}} E\left[\eta^{1-\frac{\theta}{\sigma-1}}\right] \tag{3.2}$$

As equation (3.2) demonstrates, the same level of aggregate exports can be achieved under a variety of different combinations of trade distortion parameters. As in many trade models based on a demand system with a constant elasticity of substitution, the total volume of trade is sufficient to characterize welfare. Hence, if an economy's set of parameters describing its trade regime change, real income remains the same if trade volumes stay the same. This result is summarized in Proposition 1.

Proposition 1. *The representative household in country i is indifferent between trade cost parameters $\left(\tau_{i,k}^1, f_i^{x,1}, F_i^1, G_i^1\right)$ and $\left(\tau_{i,k}^2, f_i^{x,2}, F_i^2, G_i^2\right)$ if:*

$$\left(\frac{\int t^{1-\sigma} dF_i^1(t)}{\int t^{1-\sigma} dF_i^2(t)}\right)^{\frac{\theta}{\sigma-1}} \frac{\int (f_i^{x,1}\eta)^{1-\frac{\theta}{\sigma-1}} dG_i^1(\eta)}{\int (f_i^{x,2}\eta)^{1-\frac{\theta}{\sigma-1}} dG_i^2(\eta)} = \left(\frac{\tau_{i,k}^1}{\tau_{i,k}^2}\right)^\theta \tag{3.3}$$

Proof. See Appendix D. □

Given an initial trade regime given by $\left(\tau_{i,k}^1, f_i^{x,1}, F_i^1, G_i^1\right)$, the formula in Proposition 1 describes the set of alternative trade regimes, an element of which could be $\left(\tau_{i,k}^2, f_i^{x,2}, F_i^2, G_i^2\right)$, such that trade volumes remain the same. The proof proceeds by demonstrating that, in this environment, any two trade regimes that imply the same volumes of trade imply the same prices and aggregates.

3.2 Aggregate Effects of Trade Cost Dispersion

Proposition 1 makes it easy to translate changes in idiosyncratic distortions into welfare-equivalent changes in iceberg transportation costs. This is useful because it allows for easy computation of implied changes in aggregates from changes in distortions using Proposition 2 from Arkolakis, Costinot and Rodriguez-Clare (2012). We define λ_i as one minus the import penetration ratio.[7] Then we can use the following corollary to solve for aggregates:

[7] The import penetration ratio is the ratio of imports to gross output.

Corollary 1. *Given income in the home country* Y_i^1 *and foreign country* Y_k^1, *and* λ_i^1 *associated with trade cost parameters* $\left(\tau_{i,k}^1, F_i^1, G_i^1\right)$, *then* λ_i^2 *associated with parameters* $\left(\tau_{i,k}^2, F_i^2, G_i^2\right)$ *solves:*

$$\left(\frac{\int t^{1-\sigma} dF_i^1(t)}{\int t^{1-\sigma} dF_i^2(t)}\right)^{\frac{\theta}{\sigma-1}} \frac{\int (f_i^{x,1}\eta)^{1-\frac{\theta}{\sigma-1}} dG_i^1(\eta)}{\int (f_i^{x,2}\eta)^{1-\frac{\theta}{\sigma-1}} dG_i^2(\eta)} \left(\frac{\tau_{i,k}^1}{\tau_{i,k}^2}\right)^{-\theta} = \frac{\frac{1}{1-\lambda_i^2}\frac{Y_k}{Y_i}\left(\frac{\lambda_i^1}{\lambda_i^2}\frac{1-\lambda_i^2}{1-\lambda_i^1}\right)^{\frac{1}{\varepsilon}-1} - \frac{\lambda_i^2}{\lambda_i^1}\frac{1-\lambda_i^1}{1-\lambda_i^2}}{\frac{1}{1-\lambda_i^1}\frac{Y_k}{Y_i} - 1}. \tag{3.4}$$

Proof. Follows from manipulation of the result in Proposition 2 of Arkolakis, Costinot and Rodriguez-Clare (2012) and our result in Proposition 1. □

Moreover, it is how simple to compute the change in trade volumes:

$$\frac{X_{i,k}^2}{X_{i,k}^1} = \frac{1-\lambda_i^2}{1-\lambda_i^1}\left(\frac{\lambda_i^2}{\lambda_i^1}\right)^{1/\varepsilon}. \tag{3.5}$$

4 Measuring Distortions using Firm-Level Data

In the previous section we derived relationships between sets of trade parameters. Of course, in general trade parameters are not directly observable, so the results derived so far are not easy to bring to data. In this section we show how to do just that by showing how to derive the welfare-relevant statistics that summarize the distributions of both fixed and variable trade distortions. From Proposition 1, we can see that the distribution of variable trade cost distortions can be summarized by $E[t^{1-\sigma}]$ and the distribution of fixed trade costs by $E[\eta^{\frac{\theta}{1-\sigma}}]$ for the purposes of measuring welfare.

The distribution of variable trade distortions depends crucially on variation in firm-level export intensity. Equation (2.5) gives the value of sales for firm j to a particular market. Letting $k \neq i$ be sales to the foreign market, we define $R_i(j)$ as:

$$R_i(j) \equiv \frac{p_{i,k}(j)y_{i,k}(j)}{p_{i,i}(j)y_{i,i}(j)} = \frac{Y_k P_k^\sigma}{Y_i P_i^\sigma}\left(\tau_{i,k} t_{i,k}(j)\right)^{1-\sigma}. \tag{4.1}$$

Notice that the only term on the right hand side that varies with j is $t_{i,k}$. Hence, variation in $t_{i,k}$ will be identified off of variation in R_i. However, there are two immediate impediments to doing this directly. First, the fact that $t_{i,k}$ is raised to the power $1-\sigma$ confounds the interpretation of relative variation in R_i as being attributable to variation in $t_{i,k}$. Second, there is a mechanical relationship between the size of the export market relative to the home market and export intensity that must be controlled for. One approach to address both issues is to make the auxiliary assumption that variable trade distortions are distributed log-normally, in which case both issues are relatively easy to handle by studying the distribution of the natural logarithm of export intensities. In the body of the paper we follow a general approach, but in Appendix A we derive results under that additional assumption.

To address the first issue, we show how to identify both σ and the productivity distribution parameter θ using only the coefficient of variation in domestic sales, and the aggregate trade elasticity. As an intermediate step, we derive the relationship between those two parameters as a function of the coefficient of variation in domestic sales.[8]

Lemma 1. *Let ν be the coefficient of variation of domestic sales. Then:*

$$\frac{\sigma - 1}{\theta} = \kappa \equiv \sqrt{\nu^4 + \nu^2} - \nu^2. \tag{4.2}$$

Furthermore,

$$\nu > 0 \implies \frac{\sigma - 1}{\theta} \in \left(0, \frac{1}{2}\right). \tag{4.3}$$

Proof. See Appendix E. □

Intuitively, this says that large variation in domestic sales occurs when the elasticity of substitution across goods is large, and when the tail of the productivity distribution is thick. Notice that equation (4.3) is consistent with the parameter restriction on σ and θ in Assumption 1.

To identify the productivity parameter θ, we note from equation (3.2) that the aggregate trade elasticity ε satisfies:

$$\varepsilon \equiv \frac{\partial \log\left(\frac{X_{i,k}}{X_{k,k}}\right)}{\partial \tau_{i,k}} = -\theta \text{ for } k \neq i. \tag{4.4}$$

If we take the aggregate trade elasticity as a parameter, then we can now identify the

[8]The coefficient of variation is the ratio of the standard deviation to the mean.

parameters θ and σ in closed form. In particular,

$$\theta = -\varepsilon, \text{ and } \sigma = 1 - \varepsilon\kappa. \tag{4.5}$$

Given a value of σ, we can now resolve both issues identified above to derive the welfare-relevant term summarizing the distribution of variable trade cost distortions from the distribution of firm-level export intensities.

Proposition 2. *Let $\sigma = 1 - \kappa\varepsilon$, where ε is the aggregate trade elasticity and κ is defined in Lemma 1. Then:*

$$\mu_i \equiv \frac{1}{E\left[t^{1-\sigma}\right]} = \left(\frac{E\left[R_i(j)^{\frac{\sigma}{\sigma-1}} \mid R_i(j) > 0\right]^{\frac{\sigma-1}{\sigma}}}{E\left[R_i(j) \mid R_i(j) > 0\right]} \right)^{\sigma}, \tag{4.6}$$

Proof. See Appendix F. □

The term μ_i is a sufficient statistic of the variable trade cost distribution for measuring welfare. To help interpret this equation we consider the special case where $\sigma = 2$. In that case, rearranging implies:[9]

$$\sigma = 2 \implies \mu_i = \frac{E\left[R_i(j)^2\right]}{E\left[R_i(j)\right]^2} = 1 + \frac{Var[R_i(j)]}{E\left[R_i(j)\right]^2}. \tag{4.7}$$

In this case, μ_i is a function of the square of the coefficient of variation in firm-level export intensity. Therefore, the greater the variation in export intensity, the greater is μ_i. Moreover, this measure is independent of scale effects, in the sense that a uniform increase in foreign demand has no effect on μ_i. For other values of σ, μ_i is identified by the $\sigma/(\sigma-1)$ raw moment of the distribution of R_i.

Note that the case where variable trade costs are undistorted corresponds to F_i putting all its density on $t = 1$. In this case, $\mu_i = 1$. Whenever there is dispersion in variable trade cost distortions, $\mu_i > 1$.

We identify the fixed trade cost distortions using the relationship between export status and domestic sales. In an undistorted economy, export status follows a stark productivity cutoff rule, so that every exporter is more productive than every non-

[9] The expected values and variance given here are conditional on $R_i(j) > 0$. We omit this from the equation for economy of notation.

exporter. Moreover, domestic sales are strictly increasing in productivity. Therefore, there is a strong relationship between export status and domestic sales. As variation in fixed cost distortions rises, some high productivity firms have high fixed costs of becoming exporters, and some low productivity firms have low fixed costs. Hence, the relationship between export status and productivity weakens, while the relationship between productivity and domestic sales is the same as before. Therefore, the greater is the dispersion in fixed trade cost distortions, the weaker is the relationship between domestic sales and export status. We formalize this argument in Proposition 3.

Proposition 3. *Let $I_i^x(j)$ equal one if firm j in country i is an an exporter and equal zero otherwise. Sales by firm j located in country i to its home market are given by $p_{i,i}(j)y_{i,i}(j)$. Define:*

$$\rho_i = Corr\left(I_i^x(j), p_{i,i}(j)y_{i,i}(j)\right), \tag{4.8}$$

$$\delta_i = E\left[I_i^x(j)\right]. \tag{4.9}$$

Then:

$$\rho_i = \frac{E\left[\eta^{-\frac{1}{\kappa}}\right]^{\kappa}\delta_i^{-\kappa} - 1}{\nu\sqrt{\frac{1-\delta_i}{\delta_i}}}, \tag{4.10}$$

where κ and ν are defined in Lemma 1.

Proof. See Appendix G. □

Rearranging equation (4.10) allows us to identify the welfare-relevant summary of the distribution of fixed cost distortions:

$$E\left[\eta^{\frac{\theta}{1-\sigma}}\right] = \delta_i\left(1 + \rho_i\nu\sqrt{\frac{1-\delta_i}{\delta_i}}\right)^{1/\kappa} \tag{4.11}$$

Making use of our previous results, notice that all the terms on the right hand side of this equation are observable directly in firm-level data given an aggregate trade elasticity.

Given the fraction of firms that export, this shows that the effect that dispersion in fixed trade costs has on welfare can be summarized by the correlation term, ρ_i. Unlike μ_i, clearly $\rho_i = 1$ does not correspond to the case where fixed trade costs are undistorted. Therefore, to give an interpretation to ρ_i it is useful to compare ρ_i to its undistorted baseline. Budget balance in Assumption 3 implies that fixed trade costs are undistorted if and only if G_i puts all its density on $\eta = 1$. In that case, equation (4.10) implies that the correlation when fixed costs are undistorted is:

$$E\left[\eta^{\frac{\theta}{1-\sigma}}\right] = 1 \implies \rho_i^u \equiv \frac{1-\delta_i^\kappa}{\nu\delta_i^\kappa\sqrt{\frac{1-\delta_i}{\delta_i}}}. \tag{4.12}$$

Then we can express the fixed cost distortion by comparing the correlation ρ_i to its undistorted value ρ_i^u as follows:

$$E\left[\eta^{-\frac{1}{\kappa}}\right]^\kappa = 1 + (1-\delta^\kappa)\frac{\rho_i - \rho_i^u}{\rho_i^u} = \delta^\kappa + (1-\delta^\kappa)\frac{\rho_i}{\rho_i^u} \in (0,1]. \tag{4.13}$$

Given empirical measures of the welfare-relevant expressions for the distributions of variable and fixed trade costs respectively, we can recast Proposition 1 in terms of observables.

Proposition 4. *The representative household in country i is indifferent between trade cost parameters $\left(\tau_{i,k}^1, f_i^{x,1}, F_i^1, G_i^1\right)$ and $\left(\tau_{i,k}^2, f_i^{x,2}, F_i^2, G_i^2\right)$ that imply the moments $(\mu_i^1, \rho_i^1, \delta_i^1)$ and $(\mu_i^2, \rho_i^2, \delta_i^2)$ respectively if:*

$$\frac{\tau_{i,k}^1}{\tau_{i,k}^2} = \left(\frac{\mu_i^2\delta_i^1}{\mu_i^1\delta_i^2}\frac{1+\frac{\rho_i^1}{\rho_i^{u,1}}\left[(\delta_i^1)^{-\kappa}-1\right]}{1+\frac{\rho_i^2}{\rho_i^{u,2}}\left[(\delta_i^2)^{-\kappa}-1\right]}\right)^{-\frac{1}{\kappa\varepsilon}} = \left(\frac{\mu_i^2\delta_i^1}{\mu_i^1\delta_i^2}\frac{1+\rho_i^1\nu\sqrt{\frac{1-\delta_i^1}{\delta_i^1}}}{1+\rho_i^2\nu\sqrt{\frac{1-\delta_i^2}{\delta_i^2}}}\right)^{-\frac{1}{\kappa\varepsilon}}. \tag{4.14}$$

Proof. This follows from Proposition 1 by rearranging the equation for ρ_i in Proposition 3, using the definitions of ν and κ in Lemma 1, the definition of ρ_i^u in equation (4.12), the definition of μ_i from Proposition 2, and noting that:

$$\left(\frac{f_i^{x,1}}{f_i^{x,2}}\right)^{1-\frac{\theta}{\sigma-1}} = \left(\frac{\tau_{i,k}^1}{\tau_{i,k}^2}\right)^{-\varepsilon(1-\kappa)}\left(\frac{\mu_i^2}{\mu_i^1}\right)^{1-\frac{1}{\kappa}}\left(\frac{1+\rho_i^1\nu\sqrt{\frac{1-\delta_i^1}{\delta_i^1}}}{1+\rho_i^2\nu\sqrt{\frac{1-\delta_i^2}{\delta_i^2}}}\right)^{1-\frac{1}{\kappa}}, \tag{4.15}$$

which follows from rearranging the equation for δ_i, as given in equation (G.6). □

Proposition 4 makes it simple to measure equivalent changes in iceberg trade costs for any change in our empirical measures of firm-level trade distortions. Then using Corollary 1 it is straightforward to translate changes trade costs to changes in aggregates. We demonstrate the ease with which these can be applied in the next section.

5 Application to Chinese and French Firm-level Data

In this section we apply the results from Sections 3 and 4 to firm-level data from China and France. First we show that private firms in China have lower fixed and variable trade cost distortions than the set of Chinese firms overall, and we use the results from the model to predict the effect of changing the distribution of distortions for all firms in China to that of private Chinese firms. Second we conduct a similar exercise comparing Chinese firms to French firms, and measure the gains from changing the Chinese distributions of variable and fixed trade cost distortions to those of French firms.

The results in Section 4 make it clear that distortions can be measured in any firm-level data set that has domestic and export sales together with an estimate of aggregate trade elasticities from the empirical trade literature. The effects of changing distortions on macroeconomic aggregates can be measured with the addition of total domestic income relative to total world income, and the import penetration ratio in the domestic country. We consider these data requirements to be minimal, yet they are sufficient to derive all of our results about the magnitude of distortions and the effect of changing them.

5.1 Distortions Across Firm Type in China

Our first exercise measures the effect of reducing the variable and fixed cost distortions of all Chinese firms to that of private Chinese firms. There are many legal classifications of firms in China, and many of them receive better or worse access to export markets by law based on their status. For example, firms classified as "multinational" receive privileged access to outside markets compared to other firms, while state-owned firms export very little.

We use data from the Annual Survey of Chinese Industrial Enterprises (CIE), which is conducted by the National Bureau of Statics of China (NBSC). Crucially for our exercise, this data includes information on export sales and domestic sales, as well as firm classifications that we use in our first exercise. We use only data from the year 2008, which is the latest year to which we have access. The data covers all state-owned firms, and non-state-owned firms with sales of at least 5 million RMB.

For computing changes in aggregates we make use of two other aggregate statistics to make use of Corollary 1: China's import penetration ratio and China's real income relative to its trading partner, which he take to be the rest of the world. First, we compute the import penetration ratio from values of China's gross output and imports for 2008 from National Bureau of Statistics of China (2008), which implies a value of 10.9%. Second, using Feenstra, Inklaar and Timmer (2015), we find that China accounts for 12.6% of world GDP in 2008, which implies that the GDP of the rest of the world was 6.91 times larger than China's in that year.

Based on the results in the previous two sections, we first compute the moments needed to measure the gains from reducing distortions. In the data, the coefficient of variation of domestic sales in the set of Chinese firms overall is $\nu = 1.82$, so Lemma 1 implies $\kappa = 0.47$. In our baseline results, we set the trade elasticity $\varepsilon = -4$, consistent with Simonovska and Waugh (2014), so that $\theta = 4$. By the definition of κ, this gives us $\sigma = 2.87$. In later exercises, we consider other values of ε, which will modify σ and θ according to equation (4.5).

In the Chinese data, we measure μ_i for the overall set of Chinese firms, and for the subset of Chinese firms that are private.[10] We find that μ_i is equal to 200.6 in the set of firms overall and 154.0 in the set of private firms. As is clear from Proposition 4, while the magnitude of μ_i has no direct interpretation or bearing on our results, what matters for welfare is that it is 23.2% lower for private firms than for the set of firms overall.

For the fixed cost distortion, we first measure the correlation between an indicator variable for export status and domestic sales.[11] We find that ρ_i is 0.0442 in the set

[10] We correct for observables that are likely to affect firm-level export intensity, but are not directly related to policy, by subtracting the natural logarithm of domestic sales from the natural logarithm of export sales for each firm with positive export sales, then regressing that object on controls. We then compute μ_i using the residuals from this regression. It happens that computing μ_i from the residuals or from the raw data has very little effect on our results.

[11] As with our measure of μ_i above, we again correct for observable, non-policy determinants of export status. To do this, we use the partial correlation of the indicator for export status and domestic sales rather than the raw correlation.

of firms overall, and 0.0478 for private firms. Combined with the fact that 25.87% of firms export, this allows us to identify the effect of changing the distribution of fixed cost distortions.

Our exercise is to use Proposition 4 to find the welfare-equivalent change in iceberg costs and the aggregate effects of changing μ_i and ρ_i from their values for the overall set of Chinese firms to those of private Chinese firms. In this exercise we leave the fraction of exporters, δ_i, fixed because we wish to isolate the effect of changing the dispersion of firm-level distortions. Therefore, our results can be interpreted to show the aggregate response of a change in the composition of exporters (even holding the fraction of exporting firms fixed) and the composition of sales across markets.

Table 1: Effect of Reducing Distortions to Level of Private Firms, $\varepsilon = -4$

	Equivalent Change in τ	Change in Real Income	Change in Trade
Variable Cost Distortion	−13.20%	0.81%	27.03%
Fixed Cost Distortion	−0.70%	0.03%	1.20%
	Welfare cost of moving to autarky: 2.93%		

Our baseline set of results are given in Table 1. We find that changing the distribution of variable cost distortions has a sizable effect on real income and trade volumes, but that changing the distribution of fixed cost distortions has almost no effect. The change in the variable cost distribution increases real income by 0.81% and trade volumes by 27.03%. For comparison, the table also shows the welfare cost of moving from observed levels of trade to autarky using the familiar Arkolakis, Costinot and Rodriguez-Clare (2012) formula. This shows that the model predicts that this change in μ_i would have a welfare effect that is about one quarter of the magnitude of the total gains from trade for China.

There is disagreement in the literature about the value of the trade elasticity, so next we consider cases with values higher and lower than that used in the baseline results. Table 2 shows how our results change when we change ε. As discussed above, when we vary ε, we also change σ and therefore must recompute μ_i.

The magnitude of the changes in real income realized from our two experiments change substantially as ε varies. This is intuitive, since the trade elasticity is crucial for measuring the magnitude of the real income effects from trade in this environment.

Table 2: Effect of Reducing Distortions to Level of Private Firms, varying ε

$\varepsilon = -3$	Equivalent Change in τ	Change in Real Income	Change in Trade
Variable Cost Distortion	-18.96%	1.17%	29.58%
Fixed Cost Distortion	-0.93%	0.05%	1.16%
	Welfare cost of moving to autarky: 3.92%		
$\varepsilon = -5$	Equivalent Change in τ	Change in Real Income	Change in Trade
Variable Cost Distortion	-10.01%	0.62%	25.47%
Fixed Cost Distortion	-0.56%	0.03%	1.22%
	Welfare cost of moving to autarky: 2.34%		
$\varepsilon = -10$	Equivalent Change in τ	Change in Real Income	Change in Trade
Variable Cost Distortion	-4.40%	0.27%	22.26%
Fixed Cost Distortion	-0.28%	0.01%	0.56%
	Welfare cost of moving to autarky: 1.16%		

This is clear when we note that the real income effects from these changes vary with ε in nearly the same proportion that the overall gains from trade move, as reported in the last line of each table. In economies with higher trade elasticities, trade is less valuable for real income, so it is intuitive that a policy change that increases trade volumes also has a smaller effect on real income.

5.2 Comparing Chinese and French Firms

Our second exercise is to compare trade cost distortions in Chinese firms to those in French firms. We make use of data from Amadeus for firms in the year 2008. Amadeus includes data from a number of European countries, but we focus on France because it is the only one with information on exports in that year.

Proceeding exactly as in the previous exercise, in our baseline case where $\varepsilon = -4$ we find that μ_i in the French data is 39.3. Therefore, if Chinese firms faced the distribution of variable trade cost distortions that French firms face, then μ_i in China would be 80.4% lower.[12] The correlation between export status and domestic sales, which identifies dispersion in fixed trade cost distortions, is $\rho_i = 0.2043$ in the French

[12] One possibility is that the Chinese data exhibits substantially more noise than the French data, which we readily admit could explain these results. This problem is typical in the literature on misallocation. While we cannot rule out this possibility, we note that the firms in the Chinese data are relatively large (more than 5 million RMB in sales), so that records are likely kept. Moreover, the data we use to make this calculation is only about sales, which is very likely recorded if records are being kept.

data. Again, this is substantially higher than the 0.0442 in the Chinese data.[13]

We apply Proposition 4 to measure the welfare-equivalent change in iceberg trade costs $\tau_{i,k}$ when changing μ_i and ρ_i from the values implied by the Chinese data to those of the French data. The results are reported in Table 3.

Table 3: Effect of Reducing Distortions to Level of French Firms, $\varepsilon = -4$

	Equivalent Change in τ	Change in Real Income	Change in Trade
Variable Cost Distortion	-58.18%	10.33%	303.76%
Fixed Cost Distortion	-22.11%	1.58%	52.21%
	Welfare cost of moving to autarky: 2.93%		

Given the large change in μ_i in this exercise, it is perhaps not surprising that the effects on aggregates are large. This shows that the gains to real income from moving from the Chinese to the French level of variable trade distortions are larger than the loses China would experience from moving to autarky. The gains from adopting the distribution of fixed cost distortions are also substantially larger than in the previous exercise, which follows from the fact that the increase in ρ_i was much larger.

Table 4: Effect of Reducing Distortions to Level of French Firms, varying ε

$\varepsilon = -3$	Equivalent Change in τ	Change in Real Income	Change in Trade
Variable Cost Distortion	-72.85%	16.12%	359.07%
Fixed Cost Distortion	-28.33%	2.02%	50.55%
	Welfare cost of moving to autarky: 3.92%		
$\varepsilon = -5$	Equivalent Change in τ	Change in Real Income	Change in Trade
Variable Cost Distortion	-47.62%	7.47%	273.09%
Fixed Cost Distortion	-18.12%	1.30%	53.27%
	Welfare cost of moving to autarky: 2.34%		
$\varepsilon = -10$	Equivalent Change in τ	Change in Real Income	Change in Trade
Variable Cost Distortion	-23.85%	2.97%	216.85%
Fixed Cost Distortion	-9.51%	0.69%	55.56%
	Welfare cost of moving to autarky: 1.16%		

As before, Table 4 shows how these results vary with our choice of ε. These results follow the same pattern as the previous exercise though with greater magnitudes, since this exercise implies larger changes.

[13] The implied value of the undistorted correlation in the Chinese data is $\rho_i^u = 0.2857$.

While the previous exercise pointed to the gains from changing a particular category of policy, we have not attempted in this exercise to show what particular policy changes the government of China could pursue to implement this change in the distributions of distortions faced by Chinese firms. Rather, this exercise is intended to show that gains from reducing variation in firm-level trade distortions are potentially large, and that further work is needed to understand their source.

6 Conclusion

We have shown how to measure model-implied distortions to variable and fixed trade costs in the style of Hsieh and Klenow (2009), and how to measure the aggregate effects of changing the distribution of those distortions. Applying this methodology to data from Chinese firms, we measure the gains from removing the distortions introduced by China's classification of firm types, and from moving to the levels of trade distortions exhibited by French firms.

We find that there are large gains from reducing the dispersion of trade distortions across Chinese firms. This finding implies that researchers should pay greater attention to variation in firm-level variation in trade distortions. In particular, in future work should focus on the particular ways that policies affect export propensity and export status across firms within the same country.

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A Special Case: Log-Normal Variable Trade Cost Distortions

We now consider a case in which the identification of the dispersion in variable trade cost distortions is very straightforward.

Assumption 4. *F_i is the cumulative density function of the log-normal distribution with parameters m_i and s_i.*

Notice that budget balance in Assumption 2 implies that:

$$e^{(1-\sigma)m_i+\frac{(1-\sigma)^2}{2}s_i^2} = e^{-\sigma m_i+\frac{\sigma^2}{2}s_i^2} \implies m_i = \left(\sigma - \frac{1}{2}\right)s_i^2. \tag{A.1}$$

Equation (A.1) and Assumption 4 imply that the distribution F_i can be fully characterized by s_i.

In this case, the variable cost distortion is easy to measure by simply taking the natural logarithm of the ratio of export sales to domestic sales, $R_i(j)$. Let the standard deviation of this log-ratio be called ξ_i. Under Assumption 4:

$$\xi_i = (\sigma-1)s_i \tag{A.2}$$

In this case, there is a simple formula for the variable cost distortion term given by:

$$E\left[t^{1-\sigma}\right] = e^{-\sigma\frac{\sigma-1}{2}s_i^2} = e^{-\frac{1}{2}\frac{\sigma}{\sigma-1}\xi_i^2} \tag{A.3}$$

Corollary 2. *Under Assumption 4, when G_i is held fixed Proposition 1 implies the representative household in country i is indifferent between trade cost parameters $\left(\tau_{i,k}^1, s_i^1, G_i\right)$ and $\left(\tau_{i,k}^2, s_i^2, G_i\right)$ if:*

$$\frac{\tau_{i,k}^1}{\tau_{i,k}^2} = e^{-\frac{\sigma}{2}\left[(s_i^1)^2-(s_i^2)^2\right]}. \tag{A.4}$$

Proof. Follows from equation (A.3) and Proposition 1. □

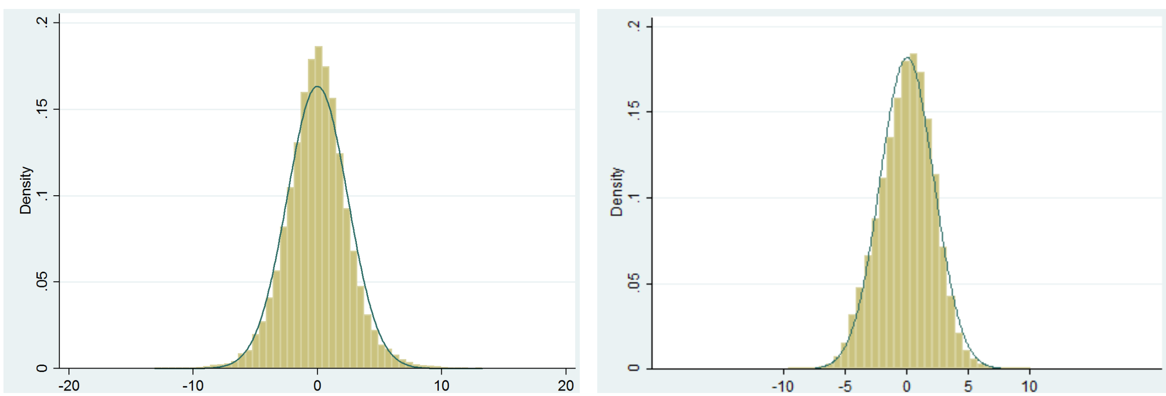
Corollary 2 and equation (A.2) allow us to express this in terms of the relationship between empirical moments and iceberg trade costs:

$$\frac{\tau_{i,k}^1}{\tau_{i,k}^2} = e^{-\frac{1}{2}\frac{\sigma}{(\sigma-1)^2}\left[(\xi_i^1)^2-(\xi_i^2)^2\right]}. \tag{A.5}$$

Figure 1: Natural Logarithm of Firm-level Export Intensity

(a) Chinese Firms

(b) French Firms



For the cases considered in Section 5, we find some support for the assumption of log-normality. We plot the natural logarithm of the ratio of export to domestic sales for in Figure 1 with normal distributions fitted over them. By inspection, in both data sets these objects seem close to normally distributed. Indeed, using Corollary 2 instead of Proposition 4 to derive our quantitative results yields very close to the same results.

B Special Case: Fixed Cost Distortions are Pareto Distributed

By their nature, fixed cost distortions are difficult to identify and our main results rely on only minimal assumptions about their distribution. However, for illustrative purposes it is useful to discuss a special case in which we assume that η follows a Pareto distribution. As in the case of log-normal variable cost distortions, this allows us to show how changes in parameters governing the dispersion of the fixed cost distortion affects our results. A Pareto distribution is a convenient example because it has a strictly positive lower bound, which is necessary to satisfy the second part of Assumption 3.

Assumption 5. *G_i is the cumulative density function of the bounded Pareto distribution with shape parameter γ_i, and scale parameter $1/a_i$ for $a_i \geq 1$.*

To satisfy budget balance in the sense of Assumption 3 this distribution must satisfy:

$$\frac{1}{a_i} = 1 - \frac{1}{\gamma_i + \frac{\theta}{\sigma-1}} \tag{B.1}$$

This implies that the distribution over fixed cost distortions can be fully characterized by a_i. Higher values of a_i are associated with greater distortions because:

$$E\left[\eta^{\frac{\theta}{1-\sigma}}\right] = a_i^{\frac{\theta}{\sigma-1}} \left(1 - \left(1 - \frac{1}{a_i}\right) \frac{\theta}{\sigma - 1}\right), \tag{B.2}$$

so it is straightforward to show that, for $a_i > 1$:

$$\frac{\partial}{\partial a_i} \left(E\left[\eta_i(j)^{\frac{\theta}{1-\sigma}}\right]\right) < 0. \tag{B.3}$$

Fixed costs are undistorted when $a_i = 1$.

Corollary 3. *Under Assumption 5, when F_i is held fixed Proposition 1 implies the representative household in country i is indifferent between trade cost parameters $\left(\tau_{i,k}^1, F_i, a_i^1\right)$ and $\left(\tau_{i,k}^2, F_i, a_i^2\right)$ if:*

$$\left(\frac{\tau_{i,k}^1}{\tau_{i,k}^2}\right)^{\theta} = \frac{(a_i^1)^{\frac{\theta}{\sigma-1}} \left(1 - \left(1 - \frac{1}{a_i^1}\right) \frac{\theta}{\sigma-1}\right)}{(a_i^2)^{\frac{\theta}{\sigma-1}} \left(1 - \left(1 - \frac{1}{a_i^2}\right) \frac{\theta}{\sigma-1}\right)}. \tag{B.4}$$

Proof. Follows from equation (B.2) and Proposition 1. □

C Alternative Timing

Suppose that the timing is adjusted such that operating firms know their variable trade cost distortion $t_i(j)$ before they decide whether or not to export. Each of the first three assumptions is otherwise maintained. Here we derive the analogue of Proposition 1 and calculate the welfare-equivalent trade cost changes from Section 5.

In this case, the export cutoffs depend on both the fixed cost distortion η and the variable cost distortion t. The analogue of equation (2.11) in this case is:

$$\bar{z}_{i,k}(\eta, t) = \left(w_i f^x \eta \frac{\sigma}{Y_k P_k^\sigma} \right)^{\frac{1}{\sigma-1}} \frac{\sigma}{\sigma-1} w_i \tau_{i,k} t. \tag{C.1}$$

In this case, variable cost budget balance implies:

$$E\left[t^{-\theta}\right] = E\left[t^{-\theta-1}\right]. \tag{C.2}$$

The equivalent of the equation from Proposition 1 in this environment is:

$$\frac{\int t^{-\theta} dF_i^1(t)}{\int t^{-\theta} dF_i^2(t)} \frac{\int (f_i^{x,1}\eta)^{1-\frac{\theta}{\sigma-1}} dG_i^1(\eta)}{\int (f_i^{x,2}\eta)^{1-\frac{\theta}{\sigma-1}} dG_i^2(\eta)} = \left(\frac{\tau_{i,k}^1}{\tau_{i,k}^2} \right)^\theta. \tag{C.3}$$

The fixed cost distortion is given by:

$$E\left[\eta^{1-\frac{\theta}{\sigma-1}}\right] = \delta_i \left(1 + \rho_i \nu \left(\frac{1-\delta_i}{\delta_i} \right)^{0.5} \right)^{\frac{\theta}{\sigma-1}} \frac{E\left[t^{-\theta}\right]^{\frac{\theta}{\sigma-1}-1}}{E\left[t^{-\theta+\sigma-1}\right]^{\frac{\theta}{\sigma-1}}}. \tag{C.4}$$

Substituting equation (C.4) into equation (C.3) implies:

$$\left(\frac{\int t^{\varepsilon} dF_i^1(t)}{\int t^{\varepsilon} dF_i^2(t)} \frac{\int t^{\varepsilon(1-\kappa)} dF_i^2(t)}{\int t^{\varepsilon(1-\kappa)} dF_i^1(t)} \right)^{1/\kappa} \frac{\delta_i^1}{\delta_i^2} \left(\frac{1+\rho_i^1 \nu \left(\frac{1-\delta_i^1}{\delta_i^1} \right)^{0.5}}{1+\rho_i^2 \nu \left(\frac{1-\delta_i^2}{\delta_i^2} \right)^{0.5}} \right)^{1/\kappa} = \left(\frac{\tau_{i,k}^1}{\tau_{i,k}^2} \right)^{-\varepsilon}. \tag{C.5}$$

The equivalent of Proposition 2 in this environment is:

$$\mu_i = \left(\frac{E[R_i(j)^{\frac{\sigma}{\sigma-1}} | R_i(j) > 0]^{\frac{\sigma-1}{\sigma}}}{E[R_i(j) | R_i(j) > 0]} \right)^{\sigma} = \frac{E[t^{-\theta+\sigma-1}]}{E[t^{-\theta}]} \tag{C.6}$$

Moreover, we can express the fraction of firms exporting as:

$$\delta_i = \left(\frac{f_i^x Y_i P_i^\sigma}{f^d Y_k P_k^\sigma} \right)^{\frac{\theta}{1-\sigma}} E[t^{-\theta}] E[\eta^{\frac{\theta}{1-\sigma}}] \tag{C.7}$$

Putting these results together, we have that Proposition 4 goes through unchanged:

$$\left(\frac{\delta_i^1}{\delta_i^2} \frac{\mu_i^2}{\mu_i^1} \frac{1+\rho_i^1 \nu \sqrt{\frac{1-\delta_i^1}{\delta_i^1}}}{1+\rho_i^2 \nu \sqrt{\frac{1-\delta_i^2}{\delta_i^2}}} \right)^{-\frac{1}{\varepsilon\kappa}} = \frac{\tau_{i,k}^1}{\tau_{i,k}^2}. \tag{C.8}$$

Given that this result is analytically identical to the case considered in the body of the text, our results in Section 5 are unchanged.

D Proof of Proposition 1

For any two economies that differ only in the trade cost parameters $(\tau_{i,k}, f_i^x, F_i, G_i)$, we first show that the representative household in country i achieves the same welfare in the two economies if $X_{i,k}$ is the same in both economies. Then the result follows from $X_{i,k}$ being equated in both economies and application of equation (3.2).

Consider two economies that differ only in their trade cost parameters. One economy is always denoted with superscripts 1 and the other with superscripts 2. If $X_{i,k}^1 = X_{i,k}^2$ then we guess that all aggregate prices and quantities are the same in both economies. We verify this guess by making heavy use of the definition of equilibrium in Section 2.2.

We verify this by substituting cutoffs and firm decision functions given by equation (2.4) into the aggregate conditions in equations (2.15), (2.16) and (2.17).

Equation (2.4) implies:

$$P_k Y_k = \frac{\theta f^d z_{min}^\theta (\sigma-1)^\theta}{\theta-\sigma+1} (\sigma w_k)^{1-\frac{\sigma\theta}{\sigma-1}} \left(\frac{Y_k P_k^\sigma}{f^d} \right)^{\frac{\theta}{\sigma-1}} + X_{i,k}. \tag{D.1}$$

Equation (2.16) implies:

$$L_i = f^d z_{min}^\theta \left(1 - \frac{\theta(\sigma-1)}{\theta-\sigma+1} \right) (\sigma-1)^\theta \left(\frac{Y_i}{f^d} \left[\frac{P_i}{\sigma w_i} \right]^\sigma \right)^{\frac{\theta}{\sigma-1}} + \left(1 - \frac{\sigma-1}{\sigma\theta} \right) \frac{X_{i,k}}{w_i}. \tag{D.2}$$

The aggregation of output implies:

$$Y_k^{\frac{\sigma-1}{\sigma}} = \frac{\theta z_{min}^\theta}{\theta-\sigma+1} Y_k^{1-1/\sigma} P_k^{\sigma-1} \left(\frac{\sigma-1}{\sigma w_k} \right)^{\sigma-1} + \frac{Y_k^{-1/\sigma}}{P_k} X_{i,k}. \tag{D.3}$$

Trade balance implies that:

$$
\begin{aligned}
& X_{i,k} = X_{k,i} \\
= & \frac{\theta f_k^x z_{min}^\theta (\sigma-1)^\theta}{\theta-\sigma+1} (\sigma w_k)^{1-\frac{\sigma\theta}{\sigma-1}} \left(\frac{Y_i P_i^\sigma}{f_k^x}\right)^{\frac{\theta}{\sigma-1}} \tau_{k,i}^{-\theta} \left[\int t^{1-\sigma} dF_k(t)\right]^{\frac{\theta}{\sigma-1}} \left[\int \eta^{1-\frac{\theta}{\sigma-1}} dG_k(\eta)\right].
\end{aligned}
\tag{D.4}
$$

Written in this way, it is clear that none of the market clearing conditions depend on any of the parameters $(\tau_{i,k}, f_i^x, F_i, G_i)$ except to the extent that they affect the aggregate value of exports $X_{i,k}$. Therefore, any prices or aggregates that satisfy the definition of equilibrium under $(\tau_{i,k}^1, f_i^{x,1} F_i^1, G_i^1)$ also satisfy the definition of equilibrium under $(\tau_{i,k}^2, f_i^{x,2}, F_i^2, G_i^2)$ so long as $X_{i,k}^1 = X_{i,k}^2$.

Then the representative household is indifferent between two economies that differ only in $(\tau_{i,k}, f_i^x, F_i, G_i)$ so long as $X_{i,k}^1 = X_{i,k}^2$. The result in Proposition 1 follows by setting $X_{i,k}^1 = X_{i,k}^2$, substituting in equation (3.2), noting that all prices and aggregates are the same in both economies, and rearranging.

E Proof of Lemma 1

Let $z_{i,d}$ be the productivity cutoff to operate in country i. Then the mean and variance of domestic sales are proportional to:

$$
E\left[p_{i,i}(j) y_{i,i}(j)\right] = \Delta_i \frac{\theta}{\theta-\sigma+1} z_{i,d}^{\sigma-1}, \tag{E.1}
$$

$$
Var\left[p_{i,i}(j) y_{i,i}(j)\right] = \Delta_i^2 z_{i,d}^{2(\sigma-1)} \left[\frac{\theta}{\theta-2(\sigma-1)} - \frac{\theta^2}{(\theta-\sigma+1)^2}\right], \tag{E.2}
$$

where Δ_i is a function of country i prices and other parameters. Then the coefficient of variation is:

$$
\nu = \frac{\sqrt{Var\left[p_{i,i}(j) y_{i,i}(j)\right]}}{E\left[p_{i,i}(j) y_{i,i}(j)\right]} = \sqrt{\frac{1}{\theta} \frac{(\theta-\sigma+1)^2}{\theta-2(\sigma-1)} - 1} = \sqrt{\frac{(1-\kappa)^2}{1-2\kappa} - 1}. \tag{E.3}
$$

Rearranging implies:

$$
\kappa^2 + 2\nu^2 \kappa + \nu^2 = 0. \tag{E.4}
$$

Then the result is the positive root of the quadratic formula, since the negative root implies that $\kappa < 0$. The bound in (4.3) follows from l'Hôpital's Rule.

F Proof of Proposition 2

Equation (4.1) implies that

$$E\left[R_i(j)\right] = \tau_{i,x}^{1-\sigma} \frac{Y_x P_x^\sigma}{Y_i P_i^\sigma} E\left[t_{i,x}(j)^{1-\sigma}\right], \tag{F.1}$$

and that

$$E\left[R_i(j)^{\frac{\sigma}{\sigma-1}}\right]^{\frac{\sigma-1}{\sigma}} = \tau_{i,x}^{1-\sigma} \frac{Y_x P_x^\sigma}{Y_i P_i^\sigma} E\left[t_{i,x}(j)^{-\sigma}\right]^{\frac{\sigma-1}{\sigma}}. \tag{F.2}$$

Taking the ratio of equations (F.1) and (F.2) implies:

$$\frac{E\left[R_i(j)\right]}{E\left[R_i(j)^{\frac{\sigma}{\sigma-1}}\right]^{\frac{\sigma-1}{\sigma}}} = \frac{E\left[t_{i,x}(j)^{1-\sigma}\right]}{E\left[t_{i,x}(j)^{-\sigma}\right]^{\frac{\sigma-1}{\sigma}}} = E\left[t_{i,x}(j)^{1-\sigma}\right]^{1/\sigma}, \tag{F.3}$$

where the last equality follows from budget balance in Assumption 2. Raising both sides to the power σ gives the result.

G Proof of Proposition 3

We begin by solving for the moments needed to compute the correlation term ρ_i.

$$\rho_i = \frac{E\left[I_i^x p_{i,i} y_{i,i}\right] - E\left[I_i^x\right] E\left[p_{i,i} y_{i,i}\right]}{\sqrt{E\left[I_i^x\right] - E\left[I_i^x\right]^2} \sqrt{E\left[(p_{i,i} y_{i,i})^2\right] - E\left[p_{i,i} y_{i,i}\right]^2}}. \tag{G.1}$$

Noting the definitions of δ_i and ν, and dividing the numerator and denominator by the expected value of both terms yields:

$$\rho_i = \frac{\frac{E\left[I_i^x p_{i,i} y_{i,i}\right]}{\delta_i E\left[p_{i,i} y_{i,i}\right]} - 1}{\sqrt{\frac{1-\delta_i}{\delta_i} \nu}} \tag{G.2}$$

Average domestic sales are given by:

$$E\left[p_{i,i}y_{i,i}\right] = z_{i,d}^{\theta}\frac{\theta f^{d}(\sigma-1)^{\theta}}{\theta-\sigma+1}(\sigma w_i)^{1-\frac{\sigma\theta}{\sigma-1}}\left(\frac{Y_i P_i^{\sigma}}{f^{d}}\right)^{\frac{\theta}{\sigma-1}}. \tag{G.3}$$

The fraction of firms that export is:

$$\delta_i = z_{i,d}^{\theta}\left(\frac{\sigma}{\sigma-1}\tau_{i,k}\right)^{-\theta} E\left[\eta^{\frac{\theta}{1-\sigma}}\right]\left(\frac{E(t^{1-\sigma})}{w_i f^{x}}\frac{Y_i P_i^{\sigma}}{\sigma}\right)^{\frac{\theta}{\sigma-1}}. \tag{G.4}$$

The domestic production cutoff is given by:

$$z_{i,d} = \left(\frac{w_i f^{d}\sigma}{Y_i P_i^{\sigma}}\right)^{\frac{1}{\sigma-1}}\frac{\sigma}{\sigma-1}w_i. \tag{G.5}$$

Then the fraction of exporters can be written as:

$$\delta_i = \tau_{i,k}^{-\theta}\left(\frac{f^{d}Y_k P_k^{\sigma}}{f^{x}Y_i P_i^{\sigma}}\right)^{\frac{\theta}{\sigma-1}} E\left[\eta^{\frac{\theta}{1-\sigma}}\right] E\left[t^{1-\sigma}\right]^{\frac{\theta}{\sigma-1}}. \tag{G.6}$$

The mean of the interaction term is:

$$\begin{aligned} E\left[I_i^{x}p_{i,i}y_{i,i}\right] &= z_{i,d}^{\theta}\int\left[\int_{\bar{z}_i(\eta)}^{\infty} Y_i P_i^{\sigma}\left(\frac{\sigma-1}{\sigma}\frac{z}{w_i}\right)^{\sigma-1}\theta z^{-\theta-1}\right]dG_i(\eta) \\ &= z_{i,d}^{\theta}\frac{\theta Y_i P_i^{\sigma}}{\theta-\sigma+1}\tau_{i,k}^{-\theta+\sigma-1}\left(\frac{\sigma-1}{\sigma w_i}\right)^{\theta} E\left[\eta^{\frac{\theta}{1-\sigma}}\right]\left(\frac{w_i f^{x}}{E\left[t^{1-\sigma}\right]}\frac{\sigma}{Y_k P_k^{\sigma}}\right)^{1-\frac{\theta}{\sigma-1}}. \end{aligned} \tag{G.7}$$

The ratio of equations (G.3) and (G.7) is:

$$\frac{E\left[I_i^{x}p_{i,i}y_{i,i}\right]}{E\left[p_{i,i}y_{i,i}\right]} = \tau_{i,k}^{-\theta+\sigma-1}\left(\frac{Y_k P_k^{\sigma}f^{d}}{Y_i P_i^{\sigma}f^{x}}\right)^{\frac{\theta}{\sigma-1}-1} E\left[\eta^{\frac{\theta}{1-\sigma}}\right] E\left[t^{1-\sigma}\right]^{\frac{\theta}{\sigma-1}-1} = \delta_i^{1-\frac{\sigma-1}{\theta}} E\left[\eta^{\frac{\theta}{1-\sigma}}\right]^{\frac{\sigma-1}{\theta}}. \tag{G.8}$$

Therefore,

$$\rho_i = \frac{\delta_i^{\frac{1-\sigma}{\theta}} E\left[\eta^{\frac{\theta}{1-\sigma}}\right]^{\frac{\sigma-1}{\theta}} - 1}{\sqrt{\frac{1-\delta_i}{\delta_i}}\nu}. \tag{G.9}$$

Then substituting in the definition of κ yields the result.